Evaluation of Crossover operators performance in Genetic Algorithms

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Abstract— Genetic Algorithms are implemented in search and optimization techniques that were developed based on ideas and techniques from genetic and evolutionary theory. Beginning with a random population of chromosomes, a genetic algorithm chooses parents from which to generate offspring using operations like selection, crossover and mutation. Here, comparisons of 5 crossover operators that are used in genetic algorithms are performed. In performance of a genetic algorithm, crossover operator has an invaluable role. It is necessary to understand the role of the crossover operator. The purpose of this project is to compare larger set of crossover operators on the same test functions and evaluate their efficiency. It also includes evaluation of statistical tests in order to study the performance of the crossover operator.

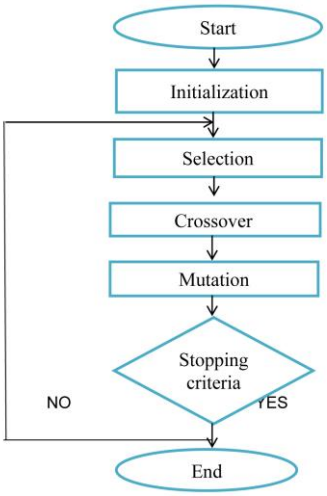
1. INTRODUCTION

A genetic algorithm (or GA) is a search technique used to find true or approximate solutions to optimization and search problems. Genetic algorithms are categorized as global search heuristics. Genetic algorithms are a particular class of evolutionary algorithms that use techniques inspired by evolutionary biology such as inheritance, mutation, selection, and crossover. The evolution usually starts from a population of randomly generated individuals. In each generation, the fitness of every individual in the population is evaluated, multiple individuals are selected from the current population (based on their fitness), and modified to form a new population. The new population is then used in the next iteration of the algorithm. Commonly, the algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population.

1.1. Initialization

Initially many individual solutions are randomly generated to form an initial population. The population size depends on the nature of the problem, but typically contains several hundreds or thousands of possible solutions. Traditionally, the population is generated randomly, covering the entire range of possible solutions. Occasionally, the solutions may be "seeded" in areas where optimal solutions are likely to be found.

1.2. Steps Involved in Genetic Algorithm



1.3. Selection

During each successive generation, a proportion of the existing population is selected to breed a new generation. Individual solutions are selected through a fitness-based process, where fitter solutions (as measured by a fitness function) are typically more likely to be selected. Certain selection methods rate the fitness of each solution and preferentially select the best solutions. Other methods rate only a random sample of the population, as this process may be very time-consuming. Most functions are stochastic and designed so that a small proportion of less fit solutions are selected. This helps keep the diversity of the population large, preventing premature convergence on poor solutions. Popular and well-studied selection methods include roulette wheel selection and tournament selection. In roulette wheel selection, individuals are given a probability of being selected that is directly proportionate to their fitness. Two individuals are then chosen randomly based on these probabilities and produce offspring.

1.4. Crossover

In genetic algorithm crossover is a genetic operator used to vary the programming of a chromosome or chromosomes from one generation to the next. Cross over is a process of taking more than one parent solutions and producing a child solution from them.

1.5. Mutation

After selection and crossover, a new population is obtained. Some are directly copied, and others are produced by crossover. In order to ensure that the individuals are not all exactly the same, mutation is performed. Mutation is, however, vital to ensuring genetic diversity within the population. Mutation occurs during evolution according to a user-definable mutation probability. This probability should be set low. If it is set too high, the search will turn into a primitive random search.

1.6. Termination

This generational process is repeated until a termination condition has been reached. Common terminating conditions are:

- A solution is found that satisfies minimum criteria
- Fixed number of generations reached
- Allocated budget (computation time/money) reached
- The highest ranking solution's fitness is reaching or has reached a plateau such that successive iterations no longer produce better results
- Manual inspection
- Any Combinations of the above

2. CROSS OVER

In the previous chapter the steps involved in Genetic algorithm was discussed in brief.Among those steps cross-over plays a vital role also our project deals with analysis of different crossover operators. In this chapter we will discuss about the crossover and its types

Crossover is a genetic operator that combines (mates) two chromosomes (parents) to produce a new chromosome (offspring). The idea behind crossover is that the new chromosome may be better

than both of the parents if it takes the best characteristics from each of the parents. Crossover occurs during evolution according to a user-definable crossover probability.

2.1. Types of Crossover

2.1.1. Single Point Crossover

A crossover operator that randomly selects a crossover point within a chromosome then interchanges the two parent chromosomes at this point to produce two new offspring.

Consider the following 2 parents which have been selected for crossover. The "|" symbol indicates the randomly chosen crossover point.

Parent1:11001|010

Parent2:00100|111

After interchanging the parent chromosomes at the crossover point, the following offspring are produced:

Offspring1:11001|111

Offspring2:00100|010

2.1.2. Two Point Crossover

A crossover operator that randomly selects two crossover points within a chromosome then interchanges the two parent chromosomes between these points to produce two new offspring. Consider the following 2 parents which have been selected for crossover. The "|" symbols indicate the randomly chosen crossover points.

Parent1:110|010|10

Parent2:001|001|11

After interchanging the parent chromosomes between the crossover points, the following offspring are produced:

Offspring1:110|001|10

Offspring2:001|010|11

2.1.3. Arithmetic Crossover

A crossover operator that linearly combines two parent chromosome vectors to produce two new offspring according to the following equations:

Offspring1 = a * Parent1 + (1- a) * Parent2

Offspring2 = (1 – a) * Parent1 + a * Parent2

where a is a random weighting factor (chosen before each crossover operation).

Consider the following 2 parents (each consisting of 4 float genes) which have been selected for crossover:

Parent 1: (0.3)(1.4)(0.2)(7.4) Parent 2: (0.5)(4.5)(0.1)(5.6)

If a = 0.7, the following two offspring would be produced:

Offspring1: (0.36)(2.33)(0.17)(6.86)

Offspring2: (0.402)(2.981)(0.149)(6.842)

2.1.4. Uniform Crossover

A crossover operator that decides which parent will contribute each of the gene values in the offspring chromosomes. This allows the parent chromosomes to be mixed at the gene level rather than the segment level. Consider the following 2 parents which have been selected for crossover:

Parent1:11001010

Parent2:00100111

If the mixing ratio is 0.5, approximately half of the genes in the offspring will come from parent 1 and the other half will come from parent 2. Below is a possible set of offspring after uniform crossover:

Offspring1: 01001110

Offspring2: 10100011

Note: The subscripts indicate which parent the gene came from.

2.1.5. Reduced Surrogate Crossover

A reduced surrogate crossover operator reduces parent strings to a skeletal form in which only those bits that differ in two parents are represented. Recombination is then limited only to positions of bits in reduced surrogates. Single point crossover was used for recombination of skeletal forms of parents. Single-point crossover operator can produce parents clones, to avoid that reduced surrogate crossover should be used. If at least one crossover point occurs between the first and last bits in reduced surrogate, then the offspring will never duplicate the parents. Also, reduced surrogate will cause that recombination process equally weight the probability of generating each offspring which can potentially be produced by an operator.

2.1.6. Heuristic Crossover

A crossover operator that uses the fitness values of the two parent chromosomes to determine the direction of the search. The offspring are created according to the following equations:

Offspring1= Best Parent + r * (Best Parent – Worst Parent)

Offspring2=Best Parent

where r is a random number between 0 and 1.

2.1.7. Multipoint Crossover

For multi-point crossover, m crossover positions number of variables of an individual, are chosen at random with no duplicates and sorted in ascending order. Then, the variables between successive crossover points are exchanged between the two parents to produce two new offspring. The section between the first variable and the first crossover point is not exchanged between individuals.

Consider the following two individuals with 11 binary variables each:

individual 1 : 0 1 1 1 0 0 1 1 0 1 0

individual 2 : 1 0 1 0 1 1 0 0 1 0 1

The chosen crossover positions are:

2 6 10

After crossover the new individuals are created:

offspring 1 : 0 1| 1 0 1 1| 0 1 1 1| 1

offspring 2 : 1 0| 1 1 0 0| 0 0 1 0| 0

2.1.8. Shuffle Crossover

Shuffle crossover is similar to one-point crossover. First, a single crossover position is selected. Before the variables are exchanged, they are randomly shuffled in both parents. After recombination, the variables in the offspring are unshuffled in reverse. This removes positional bias as the variables are randomly reassigned each time crossover is performed. In a way, shuffle crossover is similar to uniform crossover. Difference is that uniform crossover exchanges bits and not segments like shuffle crossover. Further, in uniform crossover bits exchanged follow a binary distribution and in shuffle crossover bits follow uniform distribution, as in single-point crossover.

2.1.9. Crossover Taken for Analysis

1) Single point crossover
2) Double point crossover
3) Uniform crossover
4) Shuffle crossover
5) Multipoint crossover(five points)

3. LITERATURE SURVEY

In this chapter ,we discuss about some papers which provided us some basic knowledge about cross-over operators

[1] **Wen-Yang Lin,Wen-Yuan Lee,Tzung-Pei Hong,“ Adapting Crossover And Mutation Rates In Genetic Algorithms”**

Finding optimal crossover or mutation rates vary for different problems and even for different stages of the genetic process in a problem. The crossover and the mutation rates are adapted according to the fitness values of the respective off-springs in the next generations. This paper has a genetic scheme to adapt these rates which significantly improves the performance of GA.This paper has an intuition to dynamically adjust the operators (cross-over, mutation) applied probability according to its contribution. This adjustment is made according to the measure of the performance of each operator. Adjustments are performed as follows:

CP - measures the overall performance of the cross-over operator within a generation run.

MP-measures the overall performance of the mutation operator within a generation run.

$p_c \leftarrow p_c + \theta 1$ if CP > MP
$p_c \leftarrow p_c - \theta 1$ if CP < MP
and
$p_m \leftarrow p_m + \theta 1$ if CP <MP
$p_m \leftarrow p_m - \theta 1$ if CP > MP

Where,

p_c=crossover probability, p_m=mutation probability

[2] **A.E Eiben,CoesH.M.VanKemenade, "Diagonal cross-over in GA for numerical optimization"**

This paper investigates the performance of GA if the numbers of parents are increased. Increase in number of parents in turn increases the number of crossover points. To measure this fitness value diagonal cross-over is applied. Multi parent operator creates a new value in the child chromosome based on two parents that are randomly selected. Accuracy is measured from the best objective function at the termination since ,all the objective function has a minimum value of zero.

[3] **Analysing crossover operators in Evolutionary Algorithms**

In this paper, to analyse the running time of the EAs, they derive the General Markov Cain Switching Theorem (GMCST).First the running time of the crossover operators is studied then difference between running time of crossover operators and mutations is studied. Based on that, strategies are developed. Given a particular EA and a particular problem, define phases of the optimization process and bound the time consumed in each phase, so that the total time is bounded. Hence the analysis is done case-by-case. Operators are used no matter what the current solution are. Operators however are rarely useful all the time. Thus, applying the operators only when necessary could improve the performance of the EA.

[4] **StjepanPicek, Marin Golub, and Domagoj Jakobovic ,"Evaluation of Crossover Operator Performance In GA With Binary Representation"**

In the previous studies ,the evaluation of crossover operators were made using mean and standard deviation .This didn't provide the negligible difference between the cross-over operators. To visualize these differences this paper undergoes statistical analysis.To implement this statistical approach, some test functions are taken.For each test functions every cross-over operators are applied and their performances are measured. Using this performance measure, comparison (statistically) between the operators are made and the optimum one is selected.

[5] **William M.Spears,KennethA.De Jong" An analysis of Multipoint Crossover operator"**

In this paper the efficiency of n-point crossover operator is analysed. It states that increasing the crossover points may turn into most favourable result that is the best optimal value will be reached earlier.The motivation is this paper is to extend the theoretical analysis of the cross-over operator to include multipoint variations and provde better understanding of when and how to exploit their power.

4. SYSTEM ANALYSIS

This chapter discusses about the requirements and feasibility analysis of the project. It helps to understand the viability of the project as well as basic hardware and software requirements.

4.1. Requirement Analysis

Hardware specifications

Processor	:	Intel Pentium dual core 1.73 GHz
Hard disk	:	80 GB
Memory	:	2 GB RAM

Software specifications

OS	:	Windows XP / Windows 7/8
Browser:	:	Internet Explorer 9, or Firefox 4.0
Language:	:	MATLAB

4.2. Feasibility Analysis

This is done to assess the feasibility of the project. It is used to determine if the software to be built will meet the scope and requirements of the project.

Technical feasibility

Technical feasibility analysis addresses the issues of technology used, portability and performance considerations of the technology chosen and its defects. Use of MATLAB made the implementation mathematical function easier and enhanced the performance of the project.

Financial Feasibility

These analyses the cost factor involved with the project. This project does not involve a considerable cost factor as the supporting software (MATLAB) used are open source and free to download.

Time Feasibility

This addresses the main concern whether the project can be completed on time. A detailed time plan was prepared at the time of analysis and sticking to the schedule of the project was completed within time.

Resource Feasibility

This addresses the issue of resources required to implement the project. As far as this project is concerned there was no problem with the availability and setting up of needed resources.

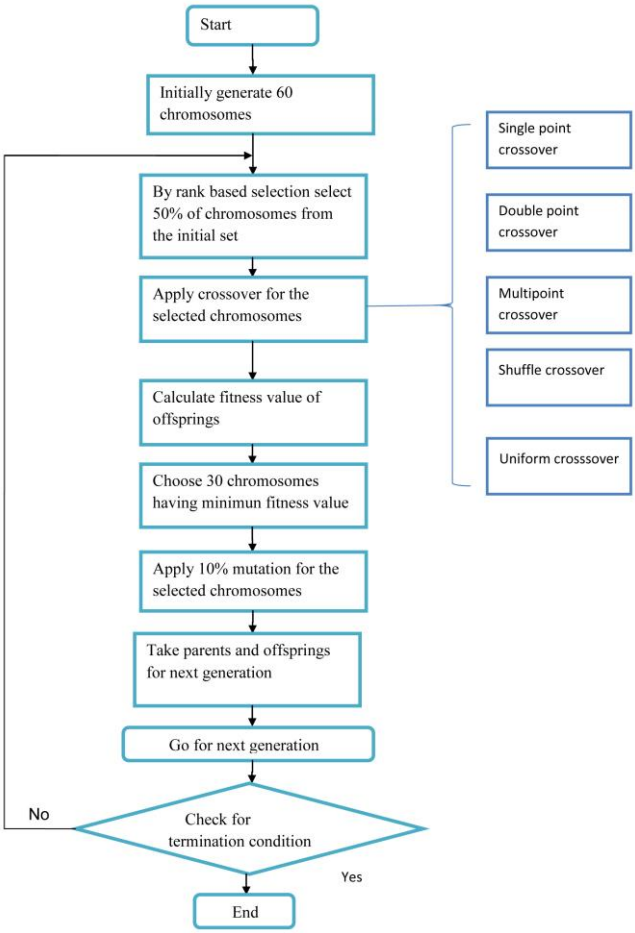
5. DESIGN AND IMPLEMENTATION

In this chapter we discuss about in detail about the design used in our project ,the parameter settings for the experiment, the implementation and analysis part of the project

Initially 60 chromosomes are taken, RANK BASED SELECTION is used to select 50% of chromosomes(30) to perform cross-over operation. In this operation 5 CROSS-OVER OPERATORS are used by one by one to analyse their performance. The children(30) generated after the cross-over operation are put to together with their parents(30) and the best 50% chromosomes(30) among the parent and the children are taken for mutation.The chromosomes before mutation(30) and chromosomes after mutation(30) are put together and taken as new population for next generation.And the generations continue until they reach the termination condition.

TABLE 1: SETUP CONDITIONS

Parameters	**Chosen**
Selection	Rank based selection
Cross-over percentage	50%
Mutation percentage	10%
Dimension of the chromosome	30
Population size	60
Generations (termination condition)	100



For analysing the performance of the cross-over operators, Genetic algorithm is applied over some optimization test functions to obtain their optimal solution. These optimization function can be mapped to some real time applications.The categories of chosen test functions are given below

5.1. Categories of Test Functions

Unimodal, convex, multidimensional: The set of functions designed for this class are easier to solve and have one global optimum. These functions can result in poor convergence to global optimum.

Multimodal and two dimensional functions with a few local optima: The set of functions under this class are of medium complexity and are serviceable to functions that have a few local optima and one global optimum.

Multimodal and two dimensional with many local optima: The set of functions under this class are more complex than the previous one as they are applicable to functions with large number of local optima.

Multimodal and multidimensional with many local optima: The functions under this set are more complex compared to the previous ones and are applied to intelligent optimization algorithms in combination with the previous class. Two dimensional problems are very rare in real world situations and hence multidimensional multimodal benchmark problems are prevalent in such situations.

5.2. Test Functions Choosen Based On the Category

De Jong function: It is a simple and a continuous benchmark problem and belongs to the first class defined above. It is mathematically defined as:

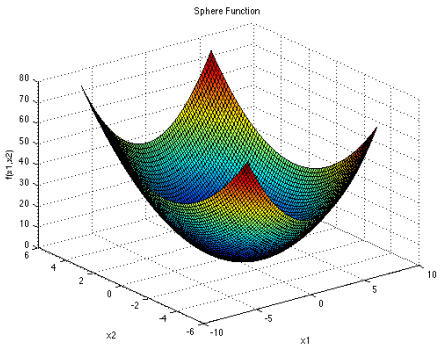


Fig. 1. Graphical representation of Dejong function

Michalewicz function: Michalewicz function is a mutimodal test function with n! local optima and the parameter m defines the difficulty of the functions, as the m goes higher the difficulty of the search increases. It is defined mathematically as:

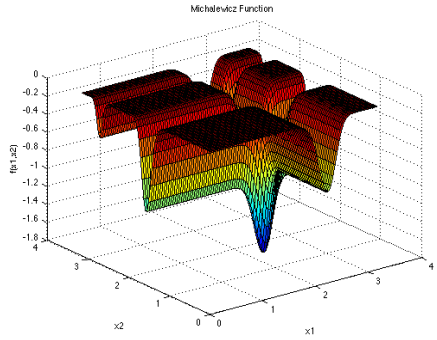


Fig. 2. Graphical representation of Michalewicz function

Rosenbrock function: It is a unimodal function and is also known as the second function of De Jong. It is defined as follows:

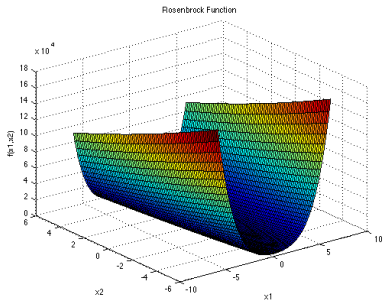


Fig. 3. Graphical representation of Rosenbrock function

Schwefel function: Schwefel function is a multimodal separable function. Global optimum for the function is distant from the next best local optima and hence the function is tricky and can drive the algorithm in a different search direction. It is defined as:

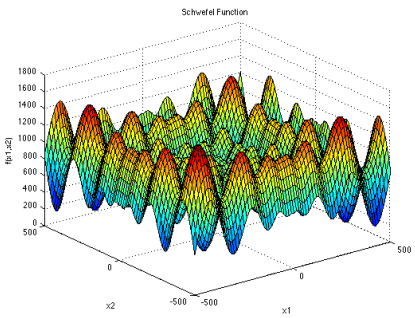


Fig. 4. Graphical representation of Schwefel function

Ackley function: Ackley is a multimodal non-separable function. It is widely used for validation and testing purposes of algorithms. It is mathematically defined as

$$f(x) = -a.\exp\left(-b.\sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n}}\right) - \exp\left(\frac{\sum_{i=1}^{n}\cos(cx_i)}{n}\right) + a + \exp(1)$$

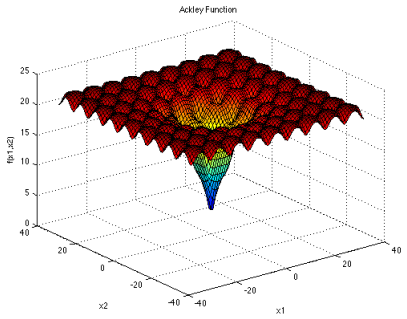


Fig. 5. Graphical representation of Ackley function

TABLE 2: RANGE OF TEST FUNCTIONS

Test functions	**Range**
Dejong	-5.12 to 5.12
Michalewicz	0 to 3.14
Rosenbrock	-2.048 to 2.048
Schwefel	-500 to 500
Ackley	-32.768 to 32.768

5.3. Implementation

- Initially one of these five functions are taken and chromosomes are generated within the range of the optimization function.
- Fitness value is calculated for all the chromosomes generated
- By using RANK BASED SELECTION the chromosomes having the best fitness value are selected to undergo cross-over operation
- cross-over operation is first done using SINGLE-POINT CROSSOVER operator,and the children are generated.Fitness values for children is also calculated.
- The fitness values of parents and chromosomes are sorted and the best 50% is chosen,the chromosomes corresponding to the best fitness values are retrieved and taken for mutation.
- Mutation percentage is set. The mutation is done only to some genes in each chromosomes according to percentage set..
- The chromosomes before mutation and chromosomes after mutation are taken as new population for next generation.
- For each generation the best fitness value is stored.
- The process continues until 100 generations are reached.
- This complete process are consider as single run ,to near the accurate value many runs are made.
- After completing multiple runs ,for each generation the average of the best fitness values that are generated in n runs is taken and plotted to analyse the performance of single point cross-over operator.

The same function is taken and the process is repeated for DOUBLE-POINT, MULTIPOINT, SHUFLLE, UNIFORM CROSS-OVER OPERATORS The results are plotted in the same graph so

that the cross-over operators can be analysed and the best cross-over operator for this function can be finalised.

The same procedure is followed for all test functions, finally the best cross-over operator for each function is found.

6. EXPERIMENTAL RESULTS

In this chapter the experimental results of the performance evaluation of cross-over operators are discussed

6.1. Dejong function

By our analysis ,Genetic algorithm implementing the cross over operation using multipoint cross-over operator converges to the best optimal solution for dejong function

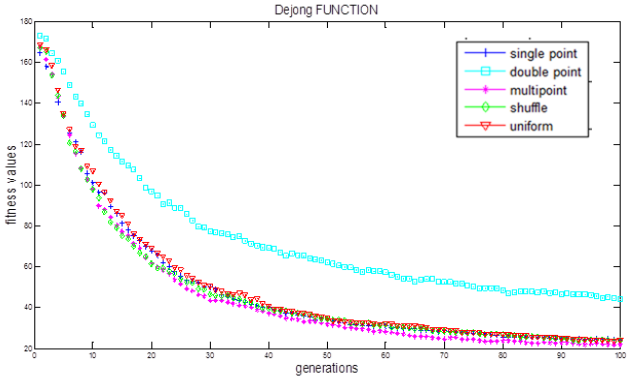


Fig. 6. Graph depicting the performance of crossover operators for Dejong function from the generations 0-100

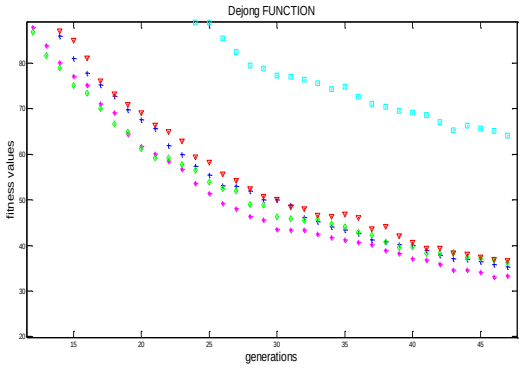


Fig. 7. Graph depicting the efficient performance of multipoint crossover from the generations 15-45

The following graph depicts clearly that multi-point converges to the best optimal solution

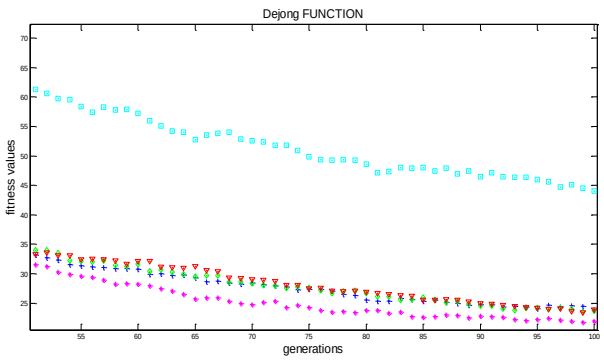


Fig. 8. Graph depicting the efficient performance of multipoint crossover from the generations 55-100

6.2. Michalewicz function

By our analysis ,Genetic algorithm implementing the cross over operation using multipoint cross-over operator converges to the best optimal solution for Michalewicz function

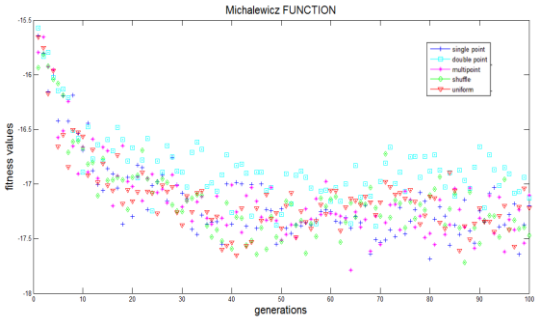


Fig. 9. Graph depicting the performance of crossover operators for Michalewicz function from the generations 0-100

The following graph depicts clearly that multi-point converges to the best optimal solution from the generations 60 to 70

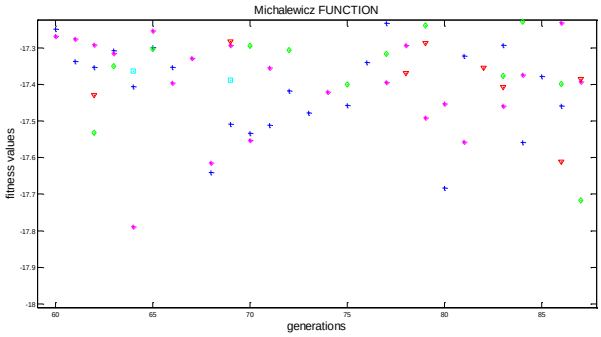


Fig. 10. Graph depicting the efficient performance of multipoint crossover from the generations 60-90

6.3. Rosenbrock function

By our analysis ,Genetic algorithm implementing the cross over operation using multipoint cross-over operator converges to the best optimal solution for Rosenbrock function

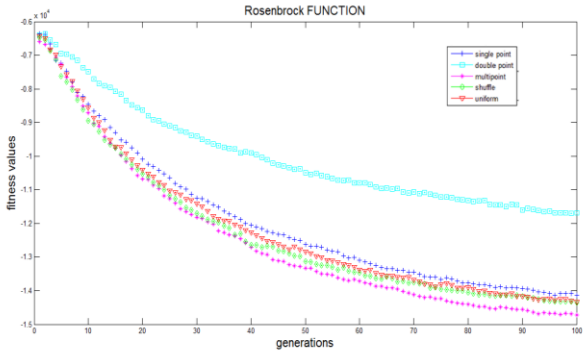


Fig. 11. Graph depicting the performance of crossover operators for Michalewicz function from the generations 0-100

The following graph depicts clearly that multi-point converges to the best optimal solution from the generations 80 to 100

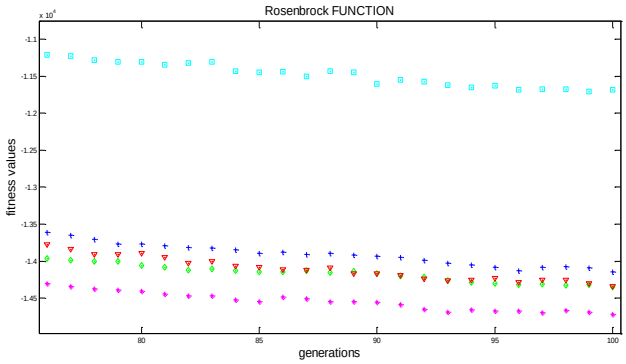


Fig. 12. Graph depicting the efficient performance of multipoint crossover from the generations 80- 100

6.4. Schwefel function

By our analysis ,Genetic algorithm implementing the cross over operation using multipoint cross-over operator converges to the best optimal solution for Schwefel function

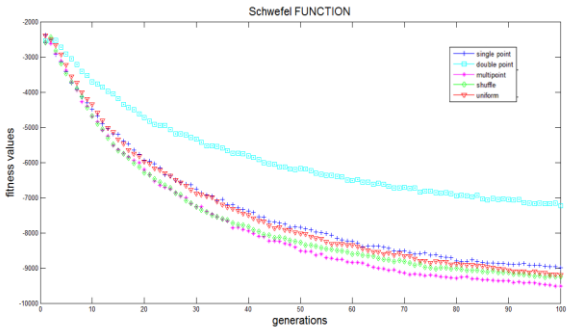


Fig. 13. Graph depicting the performance of crossover operators for Schwefel function from the generations 0-100

The following graph depicts clearly that multi-point converges to the best optimal solution from the generations 65 to 100

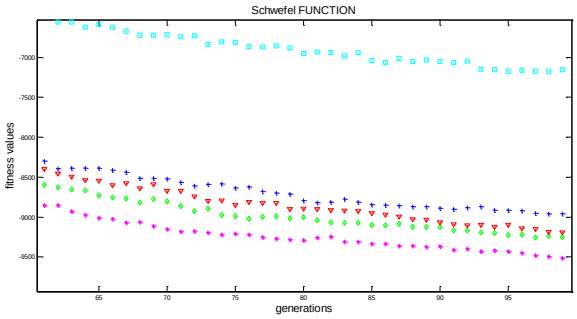


Fig. 14. Graph depicting the efficient performance of multipoint crossover from the generations 65-100

6.5. Ackley function

By our analysis ,Genetic algorithm implementing the cross over operation using multipoint cross-over operator converges to the best optimal solution for Ackley function

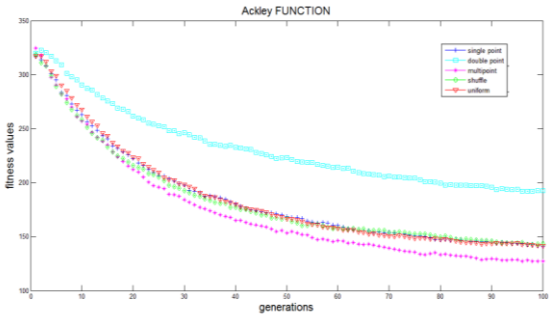


Fig. 15. Graph depicting the performance of crossover operators for Ackley function from the generations 0-100

The following graph depicts clearly that multi-point converges to the best optimal solution from the generations 65 to 95

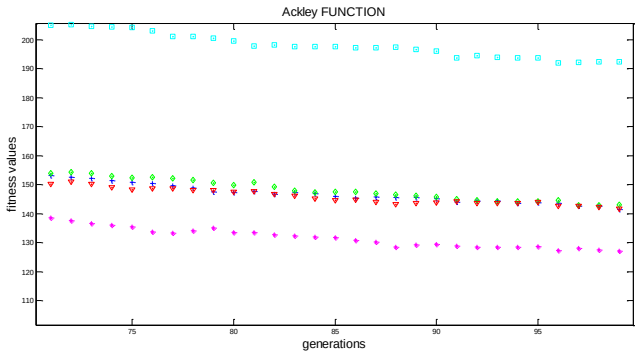


Fig. 16. Graph depicting the efficient performance of multipoint crossover from the generations 65- 95

From the above graphs it is clear that multipoint(5) crossover gives best optimum solution for all the specified test functions.So our next section makes a small analysis over the multi -point crossover operator by slightly varying the number of cross-over points.

6.6. Multi-point cross-over with 3 cross over points:

For ALL THE FIVE FUNCTIONS, when the multi-point crossover with 3 points is implemented along with the all other cross-overs the optimal solution converged is NOT BETTER than the multi-point crossover with 5 points. It is proved with following graphs

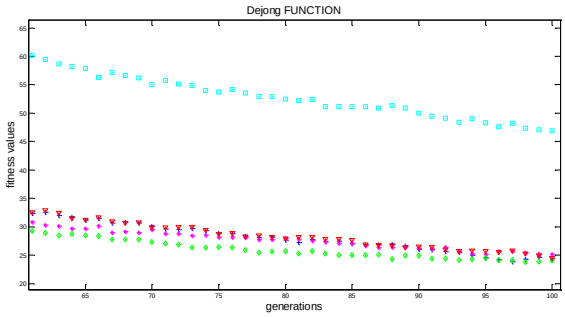


Fig. 17. Graph depicting the performance of multipoint-3 crossover operator for Dejong function

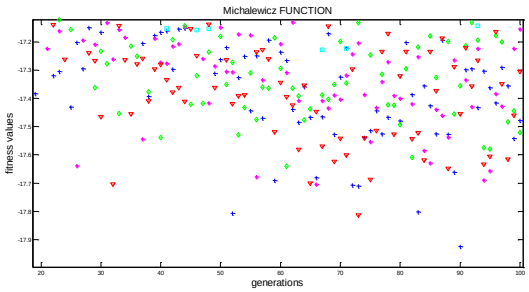


Fig. 18. Graph depicting the performance of multipoint-3 crossover operator for michalewicz function

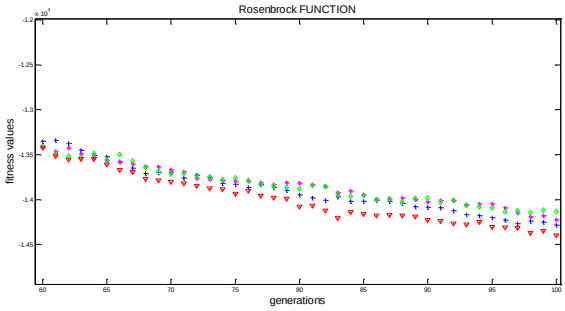


Fig. 19. Graph depicting the performance of multipoint-3 crossover operator for Rosenbrock function

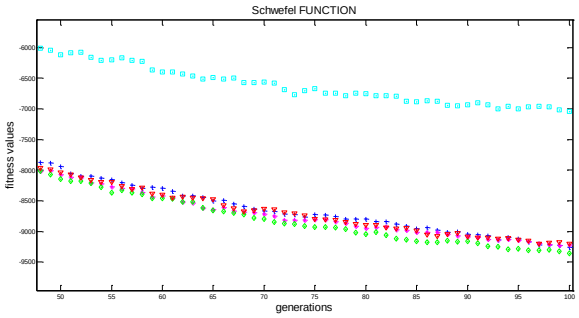


Fig. 20. Graph depicting the performance of multipoint-3 crossover operator for Schwefel function

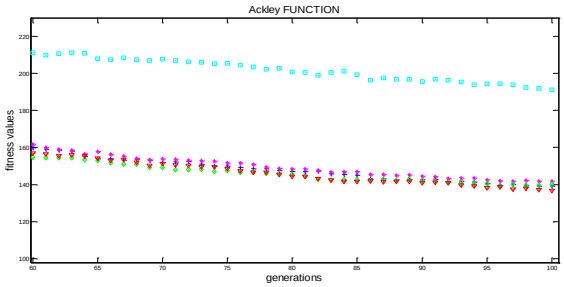


Fig. 21. Graph depicting the performance of multipoint-3 crossover operator for Ackley function

6.7. Multi-point cross-over with 7 cross over points:

For ALL THE FIVE FUNCTIONS, when the multi-point crossover with 7 points is implemented along with the all other cross-overs the optimal solution converged is BETTER than the multi-point crossover with 5 points.

It is proved with following graphs

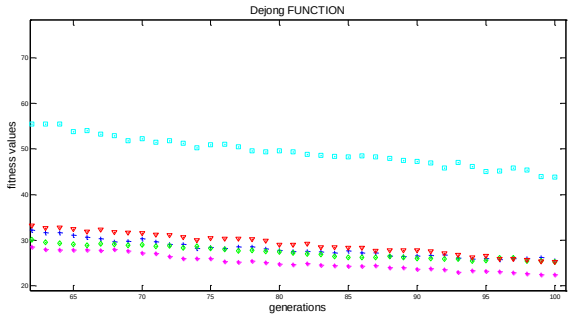


Fig. 22. Graph depicting the performance of multipoint-7 crossover operator for Dejong function

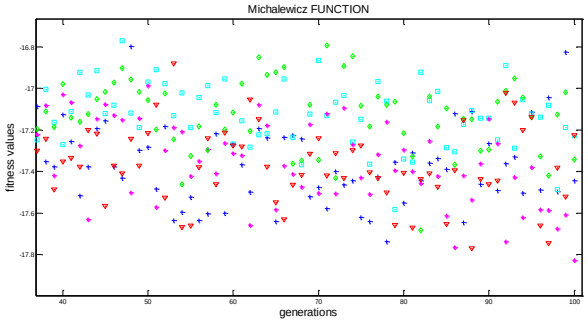


Fig. 23. Graph depicting the performance of multipoint-7 crossover operator for michalewicz function

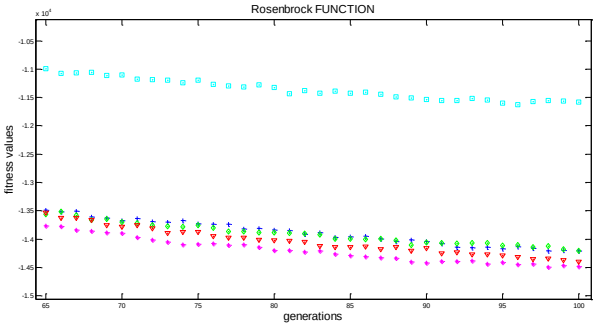


Fig. 24. Graph depicting the performance of multipoint-7 crossover operator for Rosenbrock function

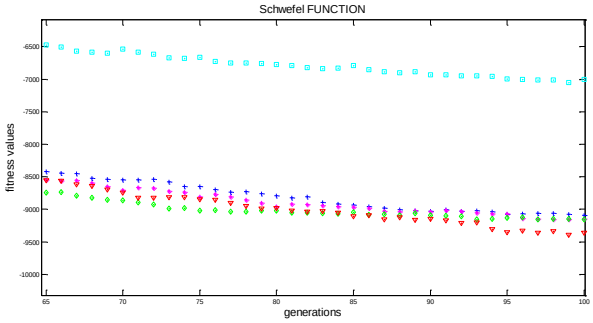


Fig. 25. Graph depicting the performance of multipoint-7 crossover operator for Schwefel function

As the analysis started with performance evaluation of five cross-over operators, multi-point cross-over with five points proved to be the efficient one. Then to analyze the multi-point with different number of cross-over points ,the analysis was done on multi-point cross-over with 3 and 7 points. In which multi-point with 7 cross-over points proved to be efficient and multi-point with 3 cross-over points proved to be inefficient.

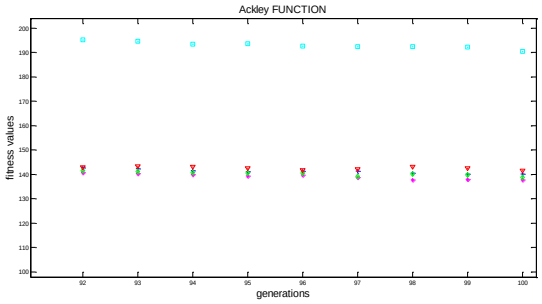


Fig. 26. Graph depicting the performance of multipoint-7 crossover operator for Ackley function

7. CONCLUSION

Thus we have analyzed a few crossover operators by applying them to five test functions. These test functions can be mapped to real-time optimization applications for which this project result will be helpful. Though all operators appears in some test runs, multipoint crossover runs for majority of runs. We conclude that multipoint crossover operator provides better performance rate.

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